Math 10A with Professor Stankova
Worksheet, Discussion #6; Friday, 9/8/2017
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Domain

Problems

1. Find the domain of $\ln(x-3)$.

> **Solution:** The domain of $\ln x$ is $(0,\infty)$. The linear shift $x-3$ shifts the function to the right by 3 and so we add 3 to the domain. This means that the domain of $\ln(x-3)$ is $(0+3,\infty+3)=(3,\infty)$.

2. Find the domain of $\sqrt{3x-3}$.

> **Solution:** The domain of $\sqrt{x}$ is $[0,\infty)$. The linear shift $3x-3$ acts by first shifting the graph right by 3, then contracting by a factor of 3. What this does to the domain is add 3 and then divide by 3, so we get $[3,\infty)$ and then $[1,\infty)$ as the final domain.

3. Find the domain of $\frac{1}{2x+1}$.

> **Solution:** The domain is when the denominator is nonzero, so when $2x+1\neq 0$ or $x\neq\frac{-1}{2}$. So the domain is $\mathbb{R}\backslash\{-1/2\}$.

4. Find the domain of $\sqrt{3-x}$.

> **Solution:** The domain is when $3-x\geq 0$ or when $x\leq 3$, meaning $(-\infty,3]$.

5. Find the domain of e^{2x+1}.

> **Solution:** The domain of e^x is everything and so shifting it to the left and right and compressing it doesn't change it. So the domain is $\mathbb{R}$.

6. Find the domain of $\sqrt{9-(2x+3)^2}$.

Solution: The domain of $\sqrt{9-x^2}$ is when $9-x^2 \geq 0$ or when $x^2 \leq 9$ or $x \in [-3,3]$. Then applying the linear shift $2x+3$ first shifts the domain to the left by 3 and then compresses it by a factor of 2. Doing so gives $[-3,3]$, then $[-6,0]$, then finally $[-3,0]$.

7. Find the domain of $\sqrt{(3x+1)^2-4}$.

Solution: The domain of $\sqrt{x^2-4}$ is when $x^2 \geq 4$ or $(-\infty,-2] \cup [2,\infty)$. The linear shift $3x+1$ shifts the domain left by 1 and then compresses it by a factor of 3. This gives $(-\infty,-3] \cup [1,\infty)$ and then $(-\infty,-1] \cup [1/3,\infty)$ as the final answer.

8. Find the domain of $\sqrt{4-(1-2x)^2}$.

Solution: The domain of $\sqrt{4-x^2}$ is $[-2,2]$. Then the linear transformation $1-2x = -2x+1$ shifts the graph left by 1 and then reflects it across the y axis and compresses it by a factor of 2. So we go from $[-2,2]$ to $[-3,1]$, to $[-1,3]$ to $[-1/2,3/2]$.

Range

Problems

9. Find the range of $2e^x+1$.

Solution: The range of e^x is $(0,\infty)$. The linear shift first stretches the range by 2 then shifts it up 1 to get $(0,\infty)$ again and then $(1,\infty)$ as the final range.

10. Find the range of $6\sin x+3$.

Solution: The range of $\sin x$ is $[-1,1]$. The range after multiplying it by 6 and adding 3 is $[-6,6]$ and then $[-3,9]$.

11. Find the range of $-2\cos x+1$.

Solution: The range of $\cos x$ is $[-1, 1]$ and multiplying by -2 gives $[-2, 2]$. Finally we add 1 to the range to get $[-1, 3]$.

12. Find the range of $5 \arctan(x) - \pi$.

Solution: The range for $\arctan(x)$ is $(-\pi/2, \pi/2)$ so the range of $5 \arctan(x) - \pi$ is $(-5\pi/2 - \pi, 5\pi/2 - \pi) = (-7\pi/2, 3\pi/2)$.

13. Find the range of $\frac{3}{x+2} - 1$.

Solution: The range of $\frac{1}{x}$ is $\mathbb{R}\backslash\{0\}$. The shift of $x+2$ changes the domain but not the range. Then multiply by 3 and subtracting 1 gives a range of $\mathbb{R}\backslash\{3 \cdot 0 - 1\} = \mathbb{R}\backslash\{-1\}$.

14. Find the range of $2 \arcsin(6x) + 1$.

Solution: The range of $\arcsin(x)$ is $[-\pi/2, \pi/2]$. Multiplying the range by 2 and adding 1 gives a range of $[-\pi + 1, \pi + 1]$.

15. Find the range of $2 - \arccos(3x + 2)$.

Solution: The range of $\arccos(x)$ is $[0, \pi]$. The linear shift $3x + 2$ shifts the domain but not the range. So the new range is $[2 - \pi, 2 - 0] = [2 - \pi, 2]$.

16. Find the range of $-4\sqrt{x} + 2$.

Solution: The range of $\sqrt{x}$ is $[0, \infty)$ so the range of $-4\sqrt{x}+2$ is $(-4\infty+2, -4\cdot 0+2] = (-\infty, 2]$.

Tangent Lines

Problems

17. Find the tangent line to $3x - 2$ at $x = -2$.

Solution: The equation is $y - y_0 = m(x - x_0)$ and so the line is $y - (3 \cdot -2 - 2) = 3(x - (-2)) \implies y = 3x - 2$.

18. Find the tangent line to $-x^2$ at $x = 1$.

Solution: Using the point slope formula with $f(x) = -x^2$ gives

$$y - f(1) = f'(1)(x - 1) \implies y + 1 = -2(x - 1) \implies y = -2x + 1.$$

19. Find the tangent line to $\frac{1}{2x}$ at $x = -4$.

Solution: We use the point slope formula with $f(x) = 1/(2x)$ and get

$$y - f(-4) = f'(-4)(x - (-4)) \implies y - (-1/8) = -1/32(x + 4) \implies y = \frac{-x}{32} - \frac{1}{4}.$$

20. Find the tangent line to x^3 at $x = -1$.

Solution: We use the point slope formula with $f(x) = x^3$ to get

$$y - f(-1) = f'(-1)(x - (-1)) \implies y - (-1) = 3(-1)^2(x + 1) \implies y = 3x + 2.$$

21. Find the tangent line to $\sqrt{x}$ at $x = 9$.

Solution: Use the point slope formula with $f(x) = \sqrt{x}$ to get

$$y - f(9) = f'(9)(x - 9) \implies y - 3 = \frac{1}{6}(x - 9) \implies y = \frac{x}{6} + \frac{3}{2}.$$

22. Find the tangent line to $6e^x$ at $x = 2$.

Solution: Use the point slope formula to get

$$y - f(2) = f'(2)(x - 2) \implies y - 6e^2 = 6e^2(x - 2) \implies y = 6e^2x - 6e^2.$$

23. Find the tangent line to $4 \cdot 7^x$ at $x = 1$.

Solution: We use the point slope formula to get

$$y - f(1) = f'(1)(x-1) \implies y - 28 = 28\ln 7(x-1) \implies y = 28x\ln 7 + 28 - 28\ln 7.$$

24. Find the tangent line to $(e^x + 2)(e^x - 2)$ at $x = 2$.

Solution: We can use the point slope formula with the expression expanded to get $f(x) = e^{2x} - 4$:

$$y - f(2) = f'(2)(x-2) \implies y - e^4 + 4 = 2e^4(x-2) \implies y = 2e^4x - 3e^4 - 4.$$

25. Find the tangent line to $\frac{x^2-9}{x-3}$ at $x = 0$.

Solution: We can simplify the function to get $\frac{x^2-9}{x-3} = x + 3$. The tangent line to a line is just that line, so $y = x + 3$.

26. Find the tangent line to $\sqrt[3]{x}/\sqrt{x}$ at $x = 1$.

Solution: We can simplify it to get $x^{-1/6}$. With this, we can calculate the tangent line with the point slope to get

$$y - 1 = \frac{-1}{6}(x-1) \implies y = \frac{-x}{6} + \frac{7}{6}.$$